Available online at www.sciencedirect.com

ScienceDirect

Procedia Computer Science 28 (2014) 285 – 293

Procedia Computer Science

Conference on Systems Engineering Research (CSER 2014)

Eds.: Azad M. Madni, University of Southern California; Barry Boehm, University of Southern California; Michael Sievers, Jet Propulsion Laboratory; Marilee Wheaton, The Aerospace Corporation
Redondo Beach, CA, March 21-22, 2014

Resilience in Large Scale Distributed Systems

Nikolai Matni*, Yoke Peng Leong, Yuh Shyang Wang, Seungil You, Matanya B. Horowitz, John C. Doyle[a]

[a]*All authors are with the California Institute of Technology, 1200 E. California Blvd., Pasadena 91125, USA*

Abstract

Distributed systems are comprised of multiple subsystems that interact in two distinct ways: (1) physical interactions and (2) cyber interactions; i.e. sensors, actuators and computers controlling these subsystems, and the network over which they communicate. A broad class of cyber-physical systems (CPS) are described by such interactions, such as the smart grid, platoons of autonomous vehicles and the sensorimotor system. This paper will survey recent progress in developing a coherent mathematical framework that describes the rich CPS "design space" of fundamental limits and tradeoffs between efficiency, robustness, adaptation, verification and scalability. Whereas most research treats at most one of these issues, we attempt a holistic approach in examining these metrics. In particular, we will argue that a control architecture that emphasizes scalability leads to improvements in robustness, adaptation, and verification, all the while having only minor effects on efficiency – i.e. through the choice of a new architecture, we believe that we are able to bring a system closer to the true fundamental hard limits of this complex design space.

Keywords: large scale; distributed; resilient; control theory; convex optimization; tradeoffs; fundamental limits; layered architecture;

* Corresponding author. Tel.: 626-590-5852; fax: 626-792-4257.
E-mail address: nmatni@caltech.edu

doi:10.1016/j.procs.2014.03.036

1. Introduction

A holistic systems theory perspective looks at the design of large-scale distributed systems through the lens of a "design space" involving fundamental tradeoffs between: (E) *Efficiency*: meeting quantitative objectives with minimal use of resources, (R) *Robustness*: maintaining efficiency despite disturbances and component uncertainty, (A) *Adaptation*: the ability to adapt to changing system components and objectives, (V) *Verification*: providing *a priori* guarantees, as formally as possible, of performance, and (S) *Scalability*: the ability to scale the design and implementation to (arbitrarily) large systems.

As it stands, although these metrics are fairly well understood individually, their combination is usually done in a serial manner, often through layering; this is especially true in aerospace and information technology related applications. Many of the great 20th century system theories in computing, control, and communications support, in limited ways, the layered separation of these objectives, but the overall process is still largely ad hoc. The goal of our work is to develop fundamental theories that aim to explain these tradeoffs between performance metrics, and in particular, that allow the designer to distinguish between which bounds are true hard limits on performance, and which are simply symptomatic of a poorly chosen architecture. In the context of biological systems, we have been able to provide a deep understanding of how R and E interact by leveraging tools from robust control theory[1], and we believe these insights to be fundamental.

In this paper, we seek to extend these insights to a particular class of distributed systems known as *cyber-physical systems* (CPS) (see Fig. 1). Such distributed systems are comprised of several subsystems that interact in two distinct but interdependent ways: (1) *physical interactions*; i.e. how each local subsystem affects its neighbors, and (2) *cyber interactions*; i.e. the sensors, actuators and computers controlling these subsystems, and the network over which they communicate and coordinate. Before delving in to how the five performance metrics that we have mentioned interact in a distributed setting, we take some time to point out some of the inherent challenges of addressing these metrics individually, and highlight some of the progress that has been made in the past decade in this respect.

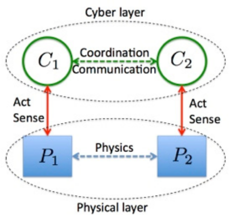


Fig. 1. A cyber-physical system is comprised of two distinct interactions: physical and cyber.

We begin with a focus on S, which is the subject of *distributed optimal control theory*. Traditional, or centralized, control theory is in general no longer applicable to large scale systems, as it assumes a central control unit that collects *all* sensor data, computes a global control action, and then broadcasts this control action out to *all* actuators. This quickly becomes both computationally intractable (the computational cost of these solutions scales poorly with the dimension of the system), and undesirable in terms of performance, as the collection of measurements/broadcasting of control actions invariably introduces *delay* in the controller. The natural solution is then to have a distributed control system, in which each local subsystem is equipped with its own controller, and in addition, delay constrained communication is allowed between these controllers: it is from this distribution of control authority that CPS emerge.

Unfortunately, in general, optimizing the performance of a distributed system can be very difficult (i.e. non-convex, and hence computationally intractable); c.f. the Witsenhausen counter-example for a canonical illustration[2]. It is only recently that a large class of systems for which the optimal synthesis problem admits a convex and hence tractable formulation, has been identified: these systems are said to satisfy a *quadratic invariance* (QI) property[3]. In the class of CPS that we consider, this condition takes a particularly simple form: it requires controllers to be able to communicate with each other faster than dynamics propagate through the physical interaction topology of the plant[4]. The intuition behind this condition is that all incentive for controllers to signal each other through the physical plant must be removed: in this way, a clear separation of the control and communication layers is maintained, leading to a tractable design problem.

When these conditions hold, it then becomes possible to synthesize optimal controllers that respect the communication constraints of the system[5, 6]. Similarly, recent results[7] are also making progress with respect to R in this setting. However, recall that the underlying motivation for distributed control is that centralized schemes are not feasible in large systems. As promising and impressive as these synthesis results have been, they ignore issues of S: despite admitting distributed implementations, they are in fact just as costly computationally as their centralized counterparts, and are often more difficult to implement. To that end, this paper presents a novel control architecture that is centered around the notion of *locality* in the closed-loop behavior of the system, and shows that this change in architecture dramatically increases S. Although theory is still being developed, through the use of a power inspired case study, we will show that this architecture has the additional important benefit of positively influencing R, A and V, all the while maintaining a high level of E. We take this to indicate that our new architecture may in fact be a step closer to achieving a holistic performance that is near the boundary of what is fundamentally achievable.

This paper is organized as follows. In Section 2, we present a minimal case study based on balancing an inverted pendulum on a person's hand. This simple example illustrates hard tradeoffs between robustness and efficiency, and formalizes the intuitive idea that delays in communication and computation can hurt overall performance. Section 3 then shifts to the distributed setting, and considers not only E and R, but S, A and V as well. In particular, we introduce our novel *localized* control architecture, and illustrate its efficacy through an LC (inductor/capacitor) circuit case study. We show that it is able to achieve the same E and R profile as a *centralized* (i.e. non S, A or V) scheme, all the while dramatically increasing S, and improving A and V. Section 4 will end with a discussion of future work, and a summary of our main points.

2. A simple case study

Balancing an inverted pendulum on a person's hand (Fig. 2) illustrates fundamental limits that constrain what robust efficiency is achievable in CPS designs *independent* of the choice of controller. The most essential issues of this problem can be illustrated experimentally by varying two parameters: the length l of the inverted pendulum, or rod, as measured from the bottom tip to the center of mass (CoM), and the point q where the person looks at the rod. Experimentally, it is observed that for most healthy adults, it is difficult to balance a shorter rod while looking at its top end, and even more difficult to balance a rod of moderate length by looking at its center point. In this section, we demonstrate how these apparently cryptic experimental results can be explained using standard sensorimotor control concepts, basic anatomy, and elementary undergraduate math, all largely independent of the highly complex underlying neural details that are still poorly understood.

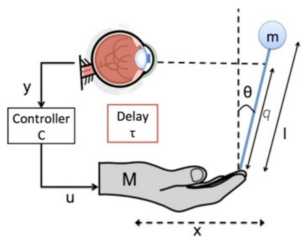


Fig. 2. A schematic of balancing an inverted pendulum on one's palm.

In this section, we illustrate essential features of CPS in an intuitive manner: namely, that physical efficiency and total CPS robustness have hard tradeoffs, and both efficiency and delays in the cyber components necessarily aggravate achievable CPS robustness. One familiar and universal feature of real CPS is that improving efficiency in terms of minimizing both the consumption and waste of material, energy, and other resources inevitably reduces the intrinsic stability of the system. Modern airplanes, rockets, communication networks, supply chains, and the future smart grid, for example, will all crash without automatic control systems. Precisely what costs in increased crash fragility are incurred for improving efficiency is what our work seeks to understand – in the case of an inverted pendulum, the answer is both elegant and intuitive.

2.1. A simple model

The standard undergraduate model of a 1-dimensional inverted pendulum on a moving cart is given by

$$\begin{aligned} &(M+m)\ddot{x} + ml(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta) = u \\ &m(\ddot{x}\cos\theta + l\ddot{\theta} - g\sin\theta) = 0 \\ &z = x + q\sin\theta, \quad y = z + n \end{aligned} \tag{0}$$

where x is the position of the cart, y is the position measurement using the eye, z is the displacement from the position of interest, and n is the sensor noise. M is the mass of the cart, m is the mass of the pendulum, g is the gravitational acceleration constant, and l and q are the CoM and measurement heights, respectively. The control objective is to maintain the absolute value of z as small as possible, as it corresponds to how far the pendulum has drifted away from the desired equilibrium.

To study local stability, we linearize the set of equations, and apply a Laplace transform: the resulting set of equations is shown in Table 1.

Table 1. Linearized time and frequency domain model.

Linearized Model	Laplace transform
$(M+m)\ddot{x} \pm ml\ddot{\theta} = u$ $m(\pm\ddot{x} + l\ddot{\theta} \mp \theta) = 0$ $z = x + q\theta$ $y = z + n$	$\begin{bmatrix} \hat{x} \\ \hat{\theta} \end{bmatrix} = \frac{1}{D(s)} \begin{bmatrix} ls^2 \mp g \\ \mp s^2 \end{bmatrix} \hat{u}$ $\hat{z} = \frac{(l-q)s^2 \mp g}{D(s)}\hat{u}, \quad \hat{y} = \hat{z} + \hat{n}$

where $D(s) = s^2(Mls^2 \mp (M+m)g)$, and the top (bottom) sign in ($\pm,\mp$) corresponds to linearization around the up (down) equilibria positions. The poles and zeros of the open loop plant are summarized in Table 2.

Table 2. Poles and zeros of an inverted pendulum on a moving cart.

Positions	Poles	Zeros		
Upright	$0, \pm\sqrt{\frac{(M+m)g}{Ml}}$	$\pm i\sqrt{\frac{g}{q-l}}$, if $q>l$,	none if $q=l$,	$\pm\sqrt{\frac{g}{l-q}}$, if $q<l$
Downward	$0, \pm i\sqrt{\frac{(M+m)g}{Ml}}$	$\pm\sqrt{\frac{g}{q-l}}$, if $q>l$,	none if $q=l$,	$\pm i\sqrt{\frac{g}{l-q}}$, if $q<l$

These well known laws say nothing about limits of robustness and efficiency, how they interact, or how they are affected by system parameters. An additional "law", known to control theorists as Bode's integral formula[8], is needed to study these relationships. This formula provides bounds for the total amplification of noise in a system, regardless of what controller is used – in this way it can provide insight into *fundamental limits* of robustness and performance. For this example in the upward position, the integral formula may be bounded as follows:

$$\frac{1}{\pi}\int_{-\infty}^{\infty}\ln|T(j\omega)|\frac{p}{p^2+\omega^2}d\omega \geq \tau p + \begin{cases} 0 & \text{if } q \geq l \\ \ln\left|\frac{z+p}{z-p}\right| & \text{if } q < l \end{cases}, \quad T(s)=\frac{(l-q)s^2 \,\square\, g}{D(s)} \tag{1}$$

where $T(s)$ is the transfer function from measurement noise n to the output z, $p=\sqrt{\frac{(M+m)g}{Ml}}$ and $z=\sqrt{\frac{g}{l-q}}$ are the unstable pole and non-minimum phase zero respectively, and τ is the total delay in the control system from measurement to actuation. As a negative value of $\ln|T(jw)|$ indicates robustness, this integral places a hard lower bound on the net achievable fragility of a control system in the presence of measurement noise. Figure 3 depicts graphically the effects of right hand plane poles, zeros and delays on the Bode integral. Areas of low sensitivity are negative, but must be balanced by the positive region where there is increased fragility. The presence of delays, unstable poles, and non-minimum phase zeros only increase the positive balance that is required.

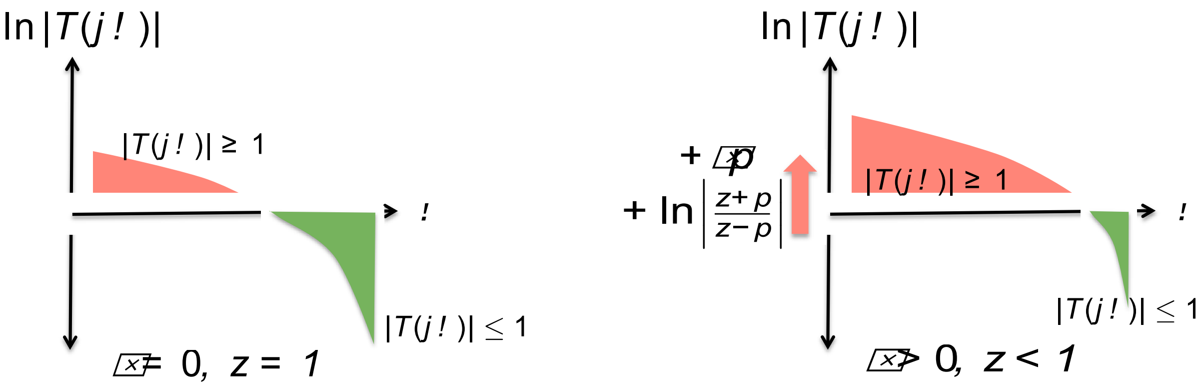


Fig. 3. An illustration of the Bode integral formula, and how right hand plane poles, zeros and delays affect it.

With this expression, we can analytically explore the impact of the parameters l, q and τ on the intrinsic fragility of the resulting system independent of the controller used: *(i) There exists a fundamental tradeoff between R and E*. Comparing the difficulty of balancing a long or short pendulum, it is easily seen that a shorter pendulum is harder to balance than a longer one. Notice that for any non-zero delay, the intrinsic fragility increases with larger p or, alternatively, with smaller l. Although highly specialized, this concept extends to more general systems, and any CPS would face similar tradeoffs, with absolute levels determined by the degree of instability (as measured by the magnitude of unstable poles) of the system, sensor noise and controller delay; *(ii) Sensing location matters*. As the eye's point of focus is moved from the tip of the pendulum to a lower point along the pendulum, the task of balancing the inverted rod becomes more and more difficult. Theoretically, this manifests in the position of the zero moving in from infinity towards the pole as q decreases. This in turn increases the bound, and hence fragility, illustrating the need to properly place sensors (and through a dual argument, actuators) in CPS systems. Fortunately, this issue can usually be easily addressed through proper design; and *(iii) Delays impact the robustness and efficiency tradeoff*. This is clearly manifest in the lower bound, as τp sets a baseline fragility. We will argue that cyber delay in communication and computing, combined with cyber-physical delay in sensing and actuation, similarly plays a large role in the achievable performance of CPS.

3. Localized distributed optimal control

The sensorimotor balancing example illustrates a universal tradeoff between robustness and efficiency with computations and experiments that can be done fairly easily. Essential to our framework is the ability for appropriate notions of robustness and efficiency to be "plugged in" as appropriate. The most severe tradeoffs in advanced CPS will always involve robustness and efficiency, and our aim in this section is to show that, through the choice of an appropriate architecture, adaptation, verification and scalability are no longer as difficult to optimize.

We will explore a simple example that illustrates what we expect to happen as we push ever closer to achievable limits in this tradeoff space, and how a new architectural approach to scalability has the potential to allow all other dimensions to be optimized in a scalable manner as well.

3.1. Control architecture

Our strategy is based on the intuitive idea that if a cyber-network's communication is faster than the speed with which the effect of a disturbance propagates through the physical layer of the plant, then the disturbance's effect can be *localized*. To achieve this, we propose a receding horizon-like control scheme in which, at each time step, every controller (i) collects estimated disturbances from a subset of its neighbors, (ii) measures its own state, (iii) computes its own estimated disturbance and broadcasts it out to a subset of its neighbors, and (iv) computes a reference trajectory and control strategy based on its collection of estimated disturbances to localize the effect of its neighbors' disturbances, as well as its own. This general strategy, as implemented at a single controller, is illustrated in Fig. 4.

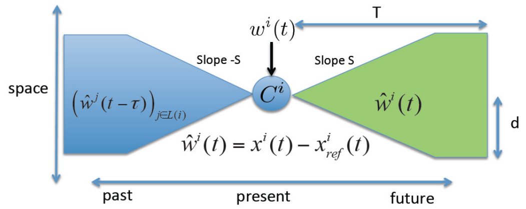


Fig. 4. Space-time cone interpretation of proposed control strategy.

Each controller can be seen as possessing a past and future "space-time" cone. The past cone represents the past information that it must collect from its neighbors in order to discern what its local disturbance is. The future cone represents those neighboring controllers that need the local controller's disturbance estimate in order to discern their own. These space-time cones are characterized by three parameters: (1) their length in time T, (2) their spatial radius d, and (3) their slope S, which is limited by communication speed. The end result of this strategy is that the effect of the local disturbance does not escape the future space-time cone.

We note that the local computation of control strategies and reference trajectories can be formulated as a convex optimization program that depends only on a local subset of the system model, the details of which we omit for the sake of brevity. As will be illustrated in the following case study, both the radius d and the length T of these space-time cones can be varied, and these variations lead to tradeoffs between locality (i.e. scalability), and performance.

3.2. LC circuit case-study

Consider an LC circuit network, as illustrated in Fig. 5. Such a circuit can be generated from an undirected graph by assigning ground capacitances to each node, and an inductance to each edge. It is arguably the simplest and most familiar example of network dynamics.

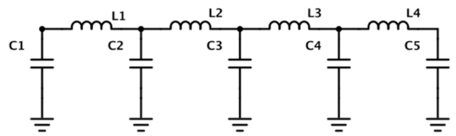


Fig. 5. A 5-node example of an LC circuit.

We set all inductances and capacitances to unity, and derive the dynamics of the circuit through a straightforward application of Kirchhoff's laws:

$$\begin{bmatrix} \dot{v} \\ \dot{i} \end{bmatrix} = \begin{bmatrix} 0 & M \\ -M^T & 0 \end{bmatrix} \begin{bmatrix} v \\ i \end{bmatrix} \tag{2}$$

where v and i are stacked vectors of node voltages and link currents, respectively, and M is the incidence matrix describing the graph. To lighten notational burden, we denote the full state of the system (stacked vector of voltages *and* currents) by x. We discretize (2) via a first order approximation with a sampling time of 0.2s, resulting in the discrete time dynamics:

$$x[k+1] = \begin{bmatrix} I & .2M \\ -.2M^T & I \end{bmatrix} x[k] =: A_d x[k] \tag{3}$$

As this is a first order approximation, it leads to an artificially unstable plant: although this is symptomatic of our discretization, it offers a convenient opportunity to illustrate the challenge of unstable networks, a growing application area. For this case study, we use a 30-node LC circuit arranged in a chain topology (this topology is illustrated in Fig. 5 for the 5-node case).

We consider actuators located on nodes in conjunction with links that are capable of changing the local voltage and current values. We define actuator density by the ratio of the number of actuators to the number of states, and for a prescribed density, we assume actuators to be uniformly distributed. In particular, we fix the density at 1/3. With these control inputs at our disposal, we may now write the full system dynamics as

$$x[k+1] = A_d x[k] + Bu[k] + w[k] \tag{4}$$

where B is a matrix of 0's and 1's, with sparsity characterized by the actuator locations, and w is an exogenous disturbance that we wish to suppress.

As alluded to earlier, when designing a controller, tradeoffs will exist, characterized in terms of efficiency and locality. In this particular case study, we will investigate how the following performance metrics tradeoff against each other: (1) efficiency (as measured in terms of state deviation and control effort), (2) communication speed/complexity, (3) locality (characterized in terms of T and d), and (4) flexibility/adaptability.

(De)centralized control concerns itself solely with criteria (1): an often used and well studied metric that allows for a principled tradeoff between state deviation and control effort is the so-called LQG cost. In order to provide a comparison with classical results, we will measure control performance with respect to the cost $||x||_2^2 + ||u||_2^2$, although our method can be used with any convex objective. We now explore some of these tradeoffs in more detail.

Communication speed: in order to illustrate the tradeoff between communication speed and performance, we consider a delayed centralized and distributed optimal controller. In the former scheme, all measurements are transmitted to a central computer that computes optimal control actions, and then broadcasts these actions to the actuators, subject to communication delays. In the distributed scheme, action is taken immediately based on locally available information and delayed global information. The performance of these two schemes is compared in Fig. 6: it becomes clear that only for unrealistically fast communication does the naïve delayed implementation's performance compare to the distributed one.

However, this performance comes at a cost: each local controller must maintain estimates of the *full* state of the system, making it difficult to apply to large systems. It becomes apparent that scalability is now a critical piece of the puzzle – we now examine the performance of our novel control architecture, and in particular investigate how locality affects other performance metrics.

Space-time cones: we begin by discussing the merits of small future space-time cones (analogous arguments can be made for past cones). A small space-time cone implies *locality* of control and coordination: each controller only needs to communicate with nodes within a small distance of itself, and consequently only needs to maintain a model of the corresponding local subset of the plant. Furthermore, locality extends to control design, allowing changes to be made without having to redesign the entire control architecture.

Of course, as is the theme of this paper, such gains invariably come at a cost: as there is a lack of coordination between neighbors in designing control actions, efficiency may degrade. Additionally, driving a state to zero in a short time may come at the expense of larger transients, once again degrading efficiency. Our experiments, however, seem to indicate that there exist favourable d and T such that these negative effects are mitigated, all the while maintaining locality. For our case study, d=8 and $T=29$ led to acceptable LQG performance, and are therefore used for comparison with other control schemes (see Fig. 6).

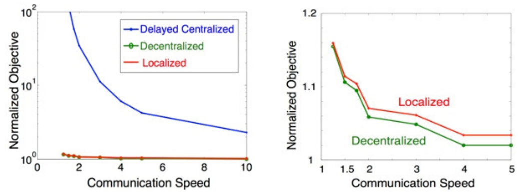


Fig. 6. Communication speed vs. efficiency for various control schemes. The right figure is a closer look at the red and green curve from the left.

In order to illustrate the (lack of) locality of the different control schemes, we plot the space-time evolution of a single disturbance hitting the middle link in Table 3. In particular, we see a concrete realization of future space time cones – notice that only the localized scheme has finite cones in both space and time.

Summary: in Table 4, we summarize the results our experiments. As can be seen, our localized control scheme can achieve similar performance to that of the centralized controller, but can be computed and implemented in a realistic, simple and *scalable* manner.

Table 3. Poles Log magnitude plots of state deviation and control effort for a given disturbance. The horizontal axis is time, the vertical space, and the legend indicates magnitude.

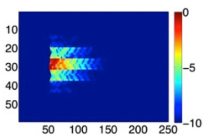
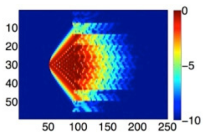
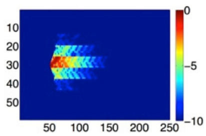
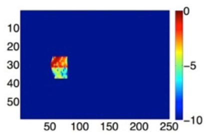
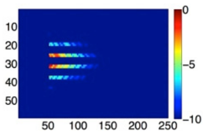
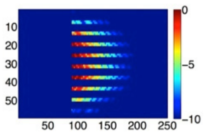
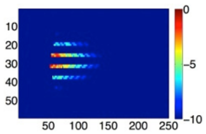
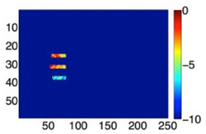
	Centralized	Delayed Centralized	Decentralized	Localized
State deviation				
Control effort				

Table 4. A comparison between different control schemes.

	Centralized	Delayed Centralized	Decentralized	Localized
Communication speed	∞	1.5x	1.5x	1.5x
Cone length T	∞	∞	∞	29
Cone radius d	Maximal (58)	Maximal (58)	Maximal (58)	8
Normalized LQG cost	1	126.8	1.1	1.1

4. Discussion and summary

4.1. Perturbations, and the benefits of locality

Our control architecture, as presented, is intrinsically robust to dynamic disturbances, such as process noise – this is what each local control design is aimed at optimizing. However, by virtue of localizing the effect of disturbances propagating, the system as a whole inherits some astonishing robustness properties to un-modeled disturbances, such as actuator or sensor failures. Say for example, that an actuator goes offline – in a typical (de)centralized setting, the entire control system would have to be redesigned to take this fact into account. This is not so in our architecture: by simply increasing the space-time cone of neighboring controllers, a localized redesign can be done in order to adapt to the controller failure.

However, the method proposed makes several assumptions that are not realistic, and future work will be aimed at relaxing them. The first of these is that states are measured perfectly, and that actuation can be delivered to arbitrary precision: these are necessary requirements for perfect localization of a disturbance's effects. In reality, sensors and actuators are noisy, and disturbances will "leak" across space-time boundaries. Fortunately, there is a well developed branch of control theory, namely robust control, that explicitly takes into account un-modeled dynamics and is able to provide stability guarantees – our next step will be to integrate these results with our control architecture.

Finally, we would like to comment on the effects of local design on global performance. In the case of the LQG norm that we considered as a performance metric in this paper, there is no loss in designing locally – this is a property of the norm, which allows us to decompose the design problem into a disturbance-by-disturbance optimization. This is not true in general however: induced norms, i.e. those that measure worst-case responses (such as H_∞) do not share this property. Future work will also explore if and how local coordination/design between controllers (particularly those that fall within the overlap of several space-time cones) can mitigate the effects of our current myopic design process under these performance metrics.

4.2. Summary

The subject of this work, a novel, localizing control architecture, represents a promising avenue for continued research in two domains. Perhaps the less obvious of the two is in the investigation of the architecture of biological systems. Such systems are tightly constrained by their physical substrates, but have nonetheless developed robust, efficient, and fault tolerant control systems. A significant contribution of this work is the ability to understand the tradeoffs that such constrained systems must make through rigorous, general analysis.

The second avenue for application of this work is in the systematic construction of distributed control systems. While the theory of centralized control systems is quite advanced and capable, that of decentralized systems is nascent. Through localization, the analysis task may now be divided between localized regions, giving hope to obtaining both theoretical hard bounds on the E, R, A, V and S space, as well as tractable computational means of achieving these bounds for systems of potentially unlimited size.

As we hope that we have already illustrated, by localizing the system, the control scheme naturally becomes scalable, robust and adaptable (with respect to distant perturbations and changes), as well as easier to verify (as this only needs to be done within each localized region). Although this is only the beginning of rigorous analysis, our numerical experiments seem to indicate that there exists an appropriate level of localization for which traditional efficiency/robustness tradeoffs are hardly affected, if at all.

It is of *fundamental* importance to be able to provide CPS designers with principled tools for understanding where the tradeoffs and hard limits lie in the massively complex design spaces of their domain. With architectural and computational tools grounded in theory, it becomes possible to understand the limits architects face and indeed to achieve these limits. We believe that the architecture that we have proposed is a step forward, bringing this goal significantly closer to reality.

References

1. Chandra, F.A., G. Buzi, and J.C. Doyle, *Glycolytic oscillations and limits on robust efficiency.* Science, 2011. **333**(6039): p. 187-192.
2. Witsenhausen, H.S., *A counterexample in stochastic optimum control.* SIAM Journal on Control, 1968. **6**(1): p. 131-147.
3. Rotkowitz, M. and S. Lall, *A Characterization of Convex Problems in Decentralized Control.* Automatic Control, IEEE Transactions on, 2006. **51**(2): p. 274-286.
4. Rotkowitz, M., R. Cogill, and S. Lall, *Convexity of optimal control over networks with delays and arbitrary topology.* Int. J. Syst., Control Commun., 2010. **2**(1/2/3): p. 30-54.
5. Lamperski, A. and J.C. Doyle, *Output Feedback H2 Model Matching for Decentralized Systems with Delays*, in *2013 IEEE American Control Conference, To appear.* 2012.
6. Mahajan, A., et al., *Information structures in optimal decentralized control*, in *Decision and Control (CDC), 2012 IEEE 51st Annual Conference on*. 2012. p. 1291-1306.
7. Scherer, C.W., *Structured Hinfinity-Optimal Control for Nested Interconnections: A State-Space Solution.* arXiv preprint arXiv:1305.1746, 2013.
8. Doyle, J.C., B.A. Francis, and A. Tannenbaum, *Feedback control theory*. Vol. 1. 1992: Macmillan Publishing Company New York.